

Technical Report

Husnu Narman

Department of Computer Science
University of Texas at San Antonio
San Antonio, TX 78249

May 06, 2011

In this report, I have studied characteristics of network and how characteristics of network affect networks topology. Specifically, I worked on how to analytically determine the topological characteristics of multi-hop wireless networks. I also developed simulations to verify the analytic formulas. The rest of this report describes my work on deriving the analytical formulas and their verification using simulations.

Topological characteristics of multi-hop wireless networks

Abstract

Understanding the topological characteristics of communications networks is essential to the design and operation of such networks in practice. Accordingly, the researchers have been considering various communications networks and their topological characteristics. In this paper, we focus on multi-hop wireless networks deployed randomly in $2D$ and $3D$ environments. The existing work in the literature provides some analytical results for a few topological characteristics using the geometrical relations in $2D$ environment. Originally, our aim was to generalize this results to $3D$ environments and to provide more analytical results for other characteristics so that we can analyze wireless networks easily and more realistically in $2D$ and $3D$ environments. However, using geometric relations in $3D$ was hard because of the excessive amount of cases. Instead of using geometrical relations, we decided to use the *distance distribution* between any two points. This is a challenging task by itself. However, if we find this distribution, then we can easily derive the same characteristics as well as others in both $2D$ and $3D$. Fortunately, the mathematicians have worked on finding such distributions and provided various important results that we use in our study. We also use simulations to verify the correctness of our analytical formulas.

1 Introduction

Wireless multi-hop networks are comprised of a collection of (mobile) nodes that communicate with each other by broadcasting radio waves within a certain distance (i.e., transmission range, r). If the distance between any two nodes is less than r , then we say there is a link between these two nodes and they can directly communicate. Otherwise, these nodes can indirectly communicate with each other through a path if there is one. Depending on the given r , the number of nodes and their locations, we can easily construct a wireless network and analyze its topological characteristics as aimed at in this paper.

Wireless nodes can be deployed in a 2D space (e.g., field, forest) or in 3D space (e.g., ocean, air). So it is important to determine their characteristics in both 2D and 3D environments. Most work in the literature considering the topological characteristics of wireless networks were done in 2D [2, 3]. These studies mainly used geometrical relations in 2D and obtained analytical formulas for *link probabilities*. These probabilities were then used to determine other characteristics such as average degree. At the beginning, we tried to find analytical formulas for link probabilities in 3D by using the same ideas in [2, 3]. However, it was difficult to analyze various cases in 3D. We also struggled to use formulas in [2, 3] to analyze other topological properties of wireless network even in 2D. After a lot of effort, we realized that if we find the *distance distribution* of random points in 2D and 3D, we can solve the problems that we encountered.

Fortunately, we found out that *distance distribution* between two random points in 2D and 3D has already been extensively studied in the literature [1, 6]. Accordingly, we use these results (specifically, the ones collectively presented in [4]) to compute link probabilities. We then use these probabilities and distance distribution to determine analytical formulas for three key topological characteristics, namely average node degree, diameter, and average path length in 2D and 3D. As part of the future work, we plan to use our approach to determine analytical formulas for other topological characteristics such as clustering coefficient, connectivity, coverage etc. To verify the correctness of our analytical formulas, we also use simulations developed in *Matlab*. We also use *Mathematica* for some symbolic calculations.

The rest of this paper is organized as follows. In Section 2 we first describe the network model and simulation environment. In Section 3 we derive the link probabilities by using distance distributions in 2D and 3D, and verify them using simulations. In Section 4, we derive the analytical formulas for the three topological characteristics that we are interested in, and verify them using simulations. Finally, we conclude this paper and give some directions for the future research in Section 6.

2 Network Model and Simulation Environment

In 2D we mainly consider the same network model used in the literature [2, 3], where multi-hop wireless network is considered as random symmetric planar points having the same transmission range, r . Also we use the same normalized setting, where the nodes are randomly deployed in a unit square of 1×1 and $0 < r \leq 1$. Similarly, in case of 3D, we will use a unit cube of $1 \times 1 \times 1$ and $0 < r \leq 1$. The underlying network N has n nodes, $v_1, v_2, v_3, v_4, \dots, v_n$ randomly placed in 1×1 or $1 \times 1 \times 1$ cube. If the distance between v_i and v_j is smaller than r , then there is a link between v_i and v_j .

In our simulations, we simply follow the above model and randomly place the nodes in unit square or cube. After positioning the nodes, we simply find the distance between every pair of nodes that placed in square or cube. If the distance between two nodes is smaller than the given r , which represents data transmission range, we put a link between them. For instance, Figure 1 shows a sample network with 50 nodes and 0.20 transmission range in 2D and 3D.

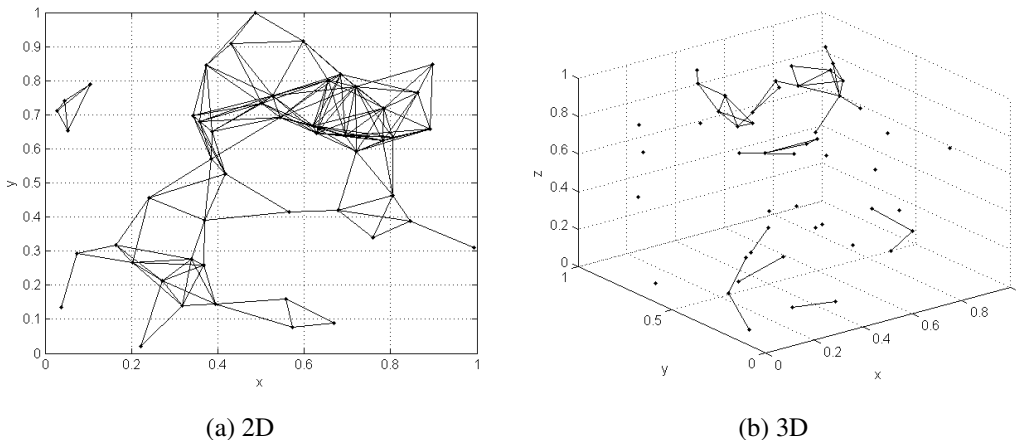


Figure 1: 2D and 3D environment with 50 nodes.

And Figure 2 shows a sample network with 50 nodes and 0.30 transmission range in 2D and 3D.

In actual simulations, we consider networks with more nodes and take the average of several replications when determining the interested topological characteristics of the simulated networks. We then compare these results with our analytical formulas. In all cases, our analytical formulas matches with the simulation results for the large number of nodes.

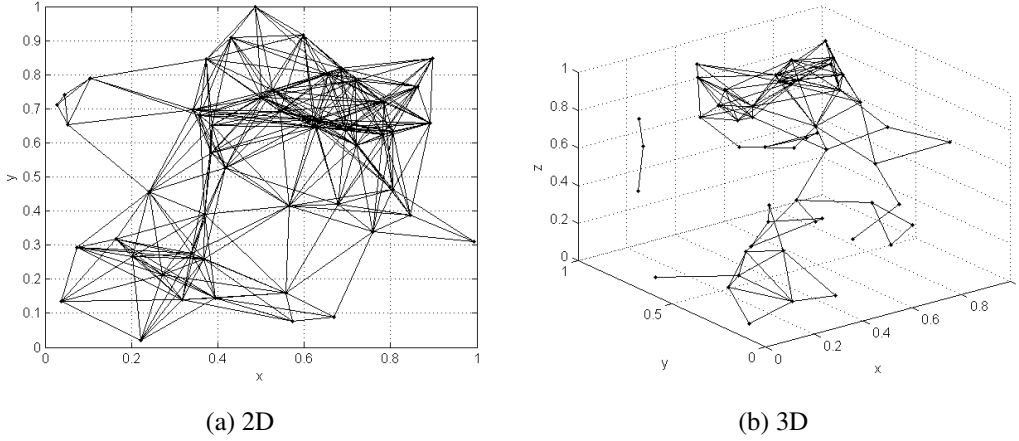


Figure 2: 2D and 3D environment with 50 nodes.

3 Link Probability

To drive analytical formulas for various topological characteristics, we first need to determine the *link probability*, which is defined as the probability that two randomly placed nodes are in communication range of each other. In other words, the probability that the distance between two points is less than or equal to r . Clearly, r is an important factor that affects the *link probability* and shapes the topology of the underlying multi-hop wireless network. For example, when r is very large, the node degrees will be big and the network will be highly connected. When r is small, the node degrees will be small and network might be partitioned into several components. So in this section, we determine the link probabilities depending on r in 2D and 3D, and verify them using simulations. In the next section, we use this probabilities to determine analytical formulas for other topological characteristics as the number of nodes increases.

3.1 Link Probability in 2D

Using geometric relations between the circle with radius r and 1×1 unit square, the authors in [2] derived the following formula for *link probability* in 2D:

$$F_{geo}(r) = \pi r^2 - \frac{8}{3}r^3 + \left(\frac{11}{3} - \pi\right)r^4, \quad (0 \leq r \leq 1) \quad (1)$$

However, as we mentioned before, finding such a formula requires to analyze several cases and hard to generalize for 3D. Also it is often possible to make mistakes due to missing overlaps between different cases. Instead of geometric relations, we consider the probability distribution function (pdf) and cumulative distribution function (cdf) of distance between two nodes randomly

positioned in unit square. Actually, this distribution has been first studied by Ghosh [1] and recently reported in [4] with several other useful pdf's and cdf's. So we mainly use the cdf presented in [4]. With our notation, the link probability that the distance between two random points in $2D$ is less than r can be given as follows:

$$F(r) = r^2 \left(\frac{r^2}{2} - \frac{8r}{3} + \pi \right), \quad (0 \leq r \leq 1) \quad (2)$$

Clearly, $F_{geo}(r)$ and $F(r)$ looks similar. However, there is a small difference as shown below:

$$\epsilon = F_{geo}(r) - F(r), \quad (0 \leq r \leq 1) \quad (3)$$

$$\epsilon = r^4 \frac{1}{42}, \quad (0 \leq r \leq 1) \quad (4)$$

We believe that $F(r)$ is more accurate than $F_{geo}(r)$ because there seems to be some overlaps that are not accurately accounted for when determining $F_{geo}(r)$.

3.2 Link Probability in 3D

The same idea of using probability distribution of distance in $2D$ can be easily generalized to $3D$ while it is hard to consider geometric relations in $3D$. To obtain *link probability* for $3D$, we have used pdf of the distance for unit cube which was found by *Piefke* [6] and again recently reported in [4]. With our notation, the pdf of distance in $3D$ would be as follows (again based on the one presented in [4]):

$$f(x) = 4\pi x^2 - 6\pi x^3 + 8x^4 - x^5, \quad (0 \leq x \leq 1) \quad (5)$$

However, this result will not give us *link probability* because we need cdf of distance between two nodes randomly positioned in unit cube. Therefore, we need to integrate equation (5) between from 0 to r to obtain *link probability* in $3D$ while r is a real number between 0 and 1. With our notation, the *link probability* that the distance between two random points in $3D$ is less than r can be given as follows:

$$F(r) = \int_0^r f(x)dx = \int_0^r (4\pi x^2 - 6\pi x^3 + 8x^4 - x^5)dx, \quad (0 \leq r \leq 1) \quad (6)$$

From this, we will have

$$F(r) = \frac{4\pi r^3}{3} - \frac{6\pi r^4}{4} + \frac{8r^5}{5} - \frac{r^6}{6}, \quad (0 \leq r \leq 1) \quad (7)$$

When dealing with *link probability*, we did not consider the part of the pdf's and cdf's accounting for the cases where the distance is greater than 1 since the distance r that we are interested in was less than 1. But when considering average path length we will consider the cases where distance is greater than 1.

3.3 Verification Using Simulations

In this section we conduct some simulations with 1000 nodes with different values of r and determined the link probabilities. We then compare these results with the analytically determined link probabilities as shown in Figure 3. Clearly, our analytical results perfectly matches with the sim-

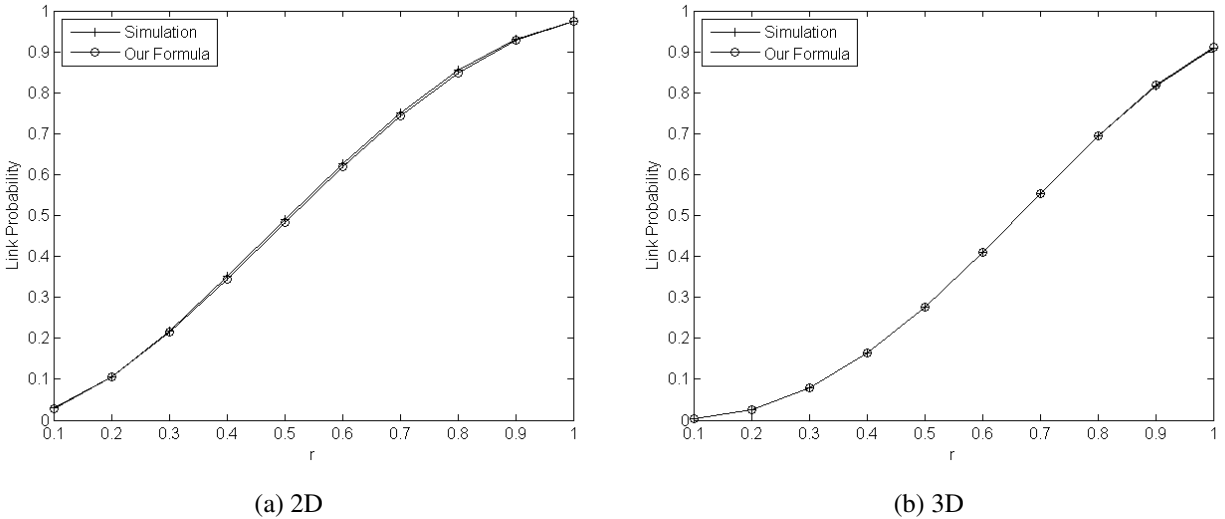


Figure 3: Comparison of link probabilities from simulations and analytical formulas.

ulation results, verifying the correctness of the analytical formulas. From the figure, we can also make a few interesting observations. The *link probability* in 2D increases faster than that in 3D and approaches to 1 as r increases. The *link probability* curve is concave upward before $r \approx 0.6$, it then becomes concave downward. This means that the increase in *link probability* before $r \approx 0.6$ is much faster than that after $r \approx 0.6$. This so-called inflection point, which can be found exactly by taking the second derivative, may have practical implications on power settings in wireless networks.

4 Topological Characteristics and Simulation Results

We consider the following three key topological characteristics here while leaving several others as the future work.

Degree: The degree of a node is the number of nodes that are within its communication range. Degree is an important characteristic as it is used to determine transmission inference. It is also important for network security and resilience. If degree of a node is high, attacking to this node can disturb the network. This problem can be seen in scale free networks.

Diameter: The maximum number of hops between any pair nodes. Knowing diameter of a network will give us information about the worst-case data transmission latency. Decreasing diameter will decrease data transmission latency while causing more collisions.

Average Path Length: The average of the shortest-path distance between nodes. This can be used to determine how efficient data transmission will be on average.

4.1 Degree

Using the *link probability* $F(r)$ for 2D and 3D from the previous section, we can find average (expected) degree as follows

$$E(x = degree) = \sum_{i=1}^{n-1} F(r) \quad (8)$$

Accordingly, the analytical formula for average degree in 2D would be

$$E(x = degree_{2D}) = (n - 1) \left(r^2 \left(\frac{r^2}{2} - \frac{8r}{3} + \pi \right) \right) \quad (9)$$

and the average degree in 3D would be

$$E(x = degree_{3D}) = (n - 1) \left(\frac{4\pi r^3}{3} - \frac{6\pi r^4}{4} + \frac{8r^5}{5} - \frac{r^6}{6} \right) \quad (10)$$

Again using simulations with 1000 nodes and varying r , we verify that the analytically determined average degree perfectly matches with the one determined by simulations, as seen in Figure 4.

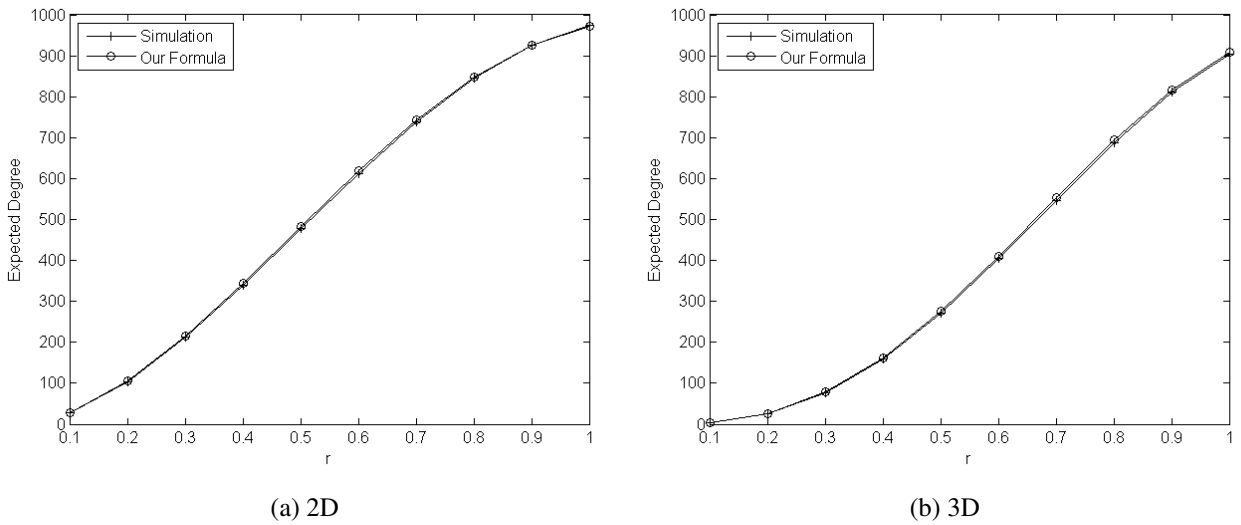


Figure 4: Comparison of average degrees from simulations and analytical formulas.

4.1.1 Diameter

In [2], it is proved that the probability that diameter will be

$$Diameter_{2D} = \left\lceil \frac{\sqrt{2}}{r} \right\rceil \quad (11)$$

as the number of nodes, n , goes to ∞ . Similarly, we can generalize this idea to more than two dimensions as follows:

$$Diameter = \left\lceil \frac{\text{length of longest diagonal line}}{\text{transmission range}} \right\rceil \quad (12)$$

as n goes to ∞ . Accordingly, the diameter in 3D would be

$$Diameter_{3D} = \left\lceil \frac{\sqrt{3}}{r} \right\rceil \quad (13)$$

as n goes to ∞ .

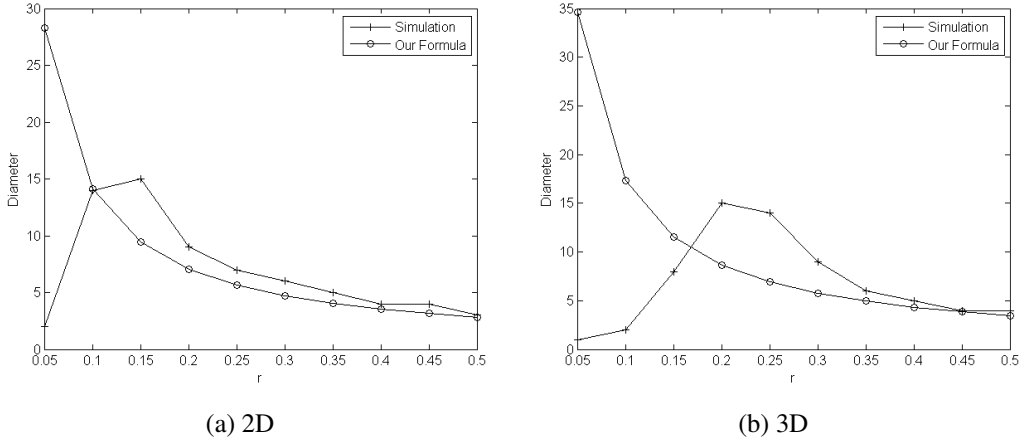


Figure 5: Comparison of diameter from simulations and analytical formulas for 100 nodes.

Figure 5 and Figure 6 represent the simulation and the analytic formulas results for 100 and 300 nodes size of networks with several transmission ranges in $2D$ and $3D$. In both $2D$ and $3D$ wireless networks, diameter formulas curve is going to exactly match to simulation curve for large r values. Although there is a pick on $3D$ figures for 300 nodes, the result for 300 nodes sized wireless network for both $2D$ and $3D$ is closer to match the simulation result for smaller r values.

5 Average Path Length

Average path length of uniformly distributed multi-hop wireless networks can be formulated approximately by using probability distribution of distance. Several probability distributions of dis-

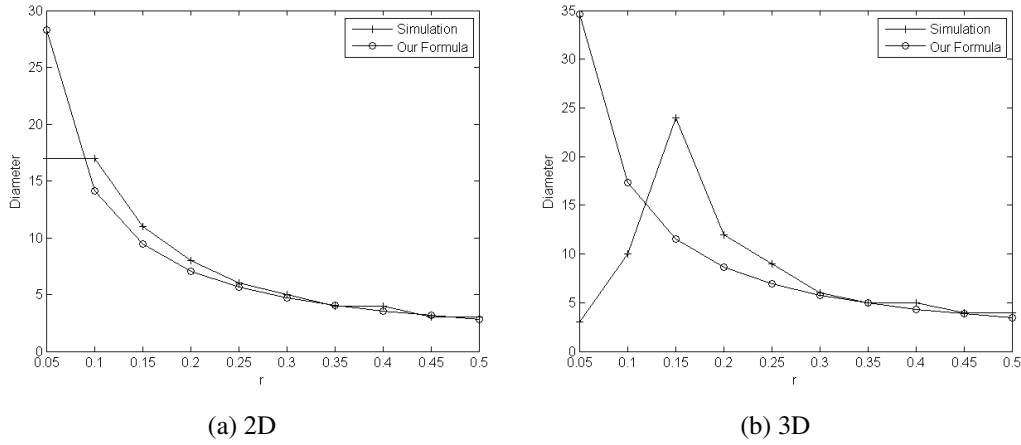


Figure 6: Comparison of diameter from simulations and analytical formulas for 300 nodes.

tance were found in literature for $2D$ and $3D$ [1] [6]. We also encounter, average length of random points can be determined by using

$$\int xf(x)dx \quad (14)$$

Therefore, we can have average number hops for the generated networks by simply

$$\frac{\int xf(x)dx}{r} \quad (15)$$

Fortunately, we have encountered that average path length was derived by Philip for $2D$ and $3D$ [5]. Average path length for $2D$ is 0.52140543 and for $3D$ is 0.661707182.

Therefore, by using (15), we will have

$$\frac{0.52140543}{r} \quad (16)$$

as the ideal average path length for $2D$ and

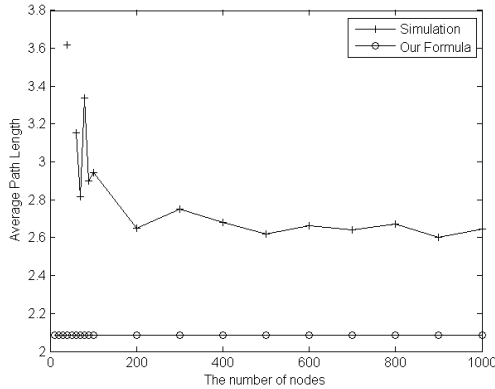
$$\frac{0.661707182}{r} \quad (17)$$

as the ideal average path length for $3D$

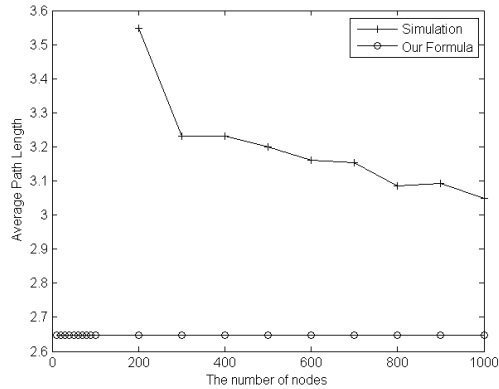
Figure 7 shows the result for average path length. Accordingly, analytical result is getting closer while when the number of nodes is increasing in $2D$ and $3D$.

6 Conclusions and Future Work

In this paper, we have used distance distribution between two random points to analyze topological characteristics of wireless networks in $2D$ and $3D$ environments. We mainly considered three



(a) 2D



(b) 3D

Figure 7: Comparison of simulated networks average path length with analytical formula result for different network size.

key topological characteristics and provided analytical formulas for them. We also conducted simulations to verified our analytical formulas. As the future work, we plan to generalize our approach and determine analytical formulas for several other topological characteristics such as connectivity, coverage, biconnectivity etc.

References

- [1] Bull Ghosh. *Calcutta Math. Soc.* 1951.
- [2] Keqin Li. Topological characteristics of random multihop wireless network. *Springer*, 8:119–216, 2005.
- [3] Gui-Lin Lu and Han-Xing Wang. Continue research on topological characteristics of random multihop wireless network. *IEEE, ICCS*, 5 2008.
- [4] A. Mazzolo. Probability density distribution of random line segments inside a convex body:application to random media. *Mathematical Physics*, 44(2), Feb 2003.
- [5] Johan Philip. The probability distribution of the distance between two random points in a box. *Mathematics Subject Classification*, 1991.
- [6] Z. Piefke. *Wahrscheinlichkeitstheor. Verwandte*, 1978.

7 Appendix

```
function prob = Link_Prob2 (x, y, r,n) % It give us the possible
count = 0; % number of links in 2D
for i=1:length(x)
    for j=i+1:length(x)
        if (r > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2))
            count=count+1;
        end
    end
end
end
prob=count/((n)*(n-1)/2);
```

```
function prob = Link_Prob3 (x, y, z, r, n) % It give us the possible
count = 0; % number of links in 3D
for i=1:length(x)
    for j=i+1:length(x)
        if (r > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2 + (z(i)-z(j))^2))
            count=count+1;
        end
    end
end
end
end
prob=count/((n)*(n-1)/2);
```

```
function f = Call_Link_Prob (n)
x = rand(1,n);
y = rand(1,n);
z = rand(1,n);
r = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9];
f = zeros(length(r));
f3 = zeros(length(r));
a = 3.14;
formula_new2d = zeros(length(r));
formula_new3d = zeros(length(r));
for i=1:length(r)
    f(i) = Link_Prob2 (x,y,r(i),n);
    f3(i) = Link_Prob3 (x,y,z,r(i),n);
```

```

    formula_new2d(i) = (a*power(r(i), 2) + ...
    ...power(r(i),4)/2 - (8.0/3)*power(r(i),3));
    formula_new3d(i) = (4*a)*power(r(i),3)/3-(6*a*power(r(i),4))/4...
    ...+8*power(r(i),5)/5-power(r(i), 6)/6;
end
figure(1);
plot (r,f,'.b');
hold on;
plot (r,formula_new2d,'or');

figure(2);
plot (r,f3,'.');
hold on
plot (r,formula_new3d,'or');

function Test_Diameter (n)

r = [0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5];
max2 = zeros(1, length(r));
max3 = zeros(1, length(r));
formula2 = zeros(1, length(r));
formula3 = zeros(1, length(r));
x = rand(1,n);
y = rand(1,n);
z = rand(1,n);
result2 = zeros(length(x), length(x));
result3 = zeros(length(x), length(x));
for s=1:length(r)
    for i=1:length(x)
        for j=1:length(x)
            if (i~=j)
                if (r(s) > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2))
                    result2(i,j) = 1;
                else
                    result2(i,j) = 2000;
                end
            end
        end
    end
end

```

```

        end
        if (r(s) > sqrt((x(i)-x(j))^2+((y(i)-y(j))^2) +(z(i)-z(j))^2))
            result3(i,j) = 1;
        else
            result3(i,j) = 2000;
        end
    end
end

end
end

for k=1:length(x)
    for i=1:length(x)
        for j=1:length(x)
            if result2(i,j) > ( result2(i,k) + result2(k,j))
                result2 (i,j) = result2(i,k) + result2(k,j);
            end
            if result3(i,j) > ( result3(i,k) + result3(k,j))
                result3 (i,j) = result3(i,k) + result3(k,j);
            end
        end
    end
end

end

for i=1:length(x)
    for j=1:length(x)
        if result2(i,j) < n
            if max2(s) < result2(i,j)
                max2(s) = result2(i,j);
            end
        end
        if result3(i,j) < n
            if max3(s) < result3(i,j)
                max3(s) = result3(i,j);
            end
        end
    end
end

```

```

        end
    end
end
for i= 1:length(r)
    formula2(i) = ceil(sqrt(2)/r(i));
    formula3(i) = ceil(sqrt(3)/r(i));
end

figure(1);
plot (r ,max2, '-+k');
hold on
plot(r, formula2,'-ok');
hleg1 = legend('Simulation','Our Formula');
set(hleg1,'Location','NorthEast')
xlabel('r');
ylabel('Diameter')

figure(2);
plot (r,max3, '-+k');
hold on
plot(r,formula3,'-ok');
hleg2 = legend('Simulation','Our Formula');
set(hleg2,'Location','NorthEast')
xlabel('r');
ylabel('Diameter')

```

```

function AvgP_Test_N (r,k)
%find the average path length of created networks and compare it the
%formula which we found from distance distrubition for 2D. We could not do
%it for 3D becasue we cannot find the formula for it because of ArcTan
%n = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200];
n = [10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600,...
.... 700, 800, 900, 1000];
A= 0;
f2 = zeros(1,length(n));

```

```

fdist2 = zeros(1,length(n));
formula_new2d = zeros(1,length(n));
x =0;
for i=1:length(n)
    for j=1:k
        [c dist]= Short_Path2(r, n(i));
        if c > 0
            A = A+ c;
            x= x+1;
        end
    end
    end
    %n(i)
    %x
    f2(i) = A/x;
    A= 0;
    x=0;
    %formula_new2d(i) = (-(8/5)+(2*pi)/3) + (1/6) *(-1 + 10*sqrt(2)...
    ... - 4 *pi + log(3 + 2 *sqrt(2)));
    formula_new2d(i) = 0.52140543/ r;
    fdist2(i) = dist/r;
end

figure(1);
plot (n ,f2, '--k');
hold on
plot(n, formula_new2d,'-ok');
hleg1 = legend('Simulation','Our Formula');
set(hleg1,'Location','NorthEast')
xlabel('The number of nodes');
ylabel('Average Path Length')

A= 0;
f3 = zeros(1,length(n));
fdist3 = zeros(1,length(n));
formula_new3d = zeros(1,length(n));
x =0;

```

```

for i=1:length(n)
    for j=1:k
        [c dist]= Short_Path3(r, n(i));
        if c > 0
            A = A+ c;
            x= x+1;
        end
    end
    end
    %n(i)
    %x
    f3(i) = A/x;
    A= 0;
    x=0;
    %formula_new2d(i) =  $(-(8/5)+(2*\pi)/3) + (1/6) *(-1 + 10*\sqrt{2})\dots$ 
    ... - 4 *pi + log(3 + 2 *sqrt(2)));
    formula_new3d(i) = 0.661707182/ r;
    fdist3(i) = dist/r;
end

```

```

figure(2);
plot (n ,f3, '--k');
hold on
plot(n, formula_new3d,'-ok');
hleg1 = legend('Simulation', 'Our Formula');
set(hleg1, 'Location', 'NorthEast')
xlabel('The number of nodes');
ylabel('Average Path Length')

```

```

function [average_path avg_dist]= Short_Path2 (r,n)
x = rand(1,n);
y = rand(1,n);
result = zeros(length(x), length(x));
result2 = zeros(length(x), length(x));
for i=1:length(x)

```



```

for j=1:length(x)
    if (i~=j)
        result2(i,j) = sqrt((x(i)-x(j))^2+(y(i)-y(j))^2);
        if (r > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2))
            result(i,j) = 1;
        else
            result(i,j) = 2000;
        end
    end
end
end

for k=1:length(x)
    for i=1:length(x)
        for j=1:length(x)
            if result(i,j) > ( result(i,k) + result(k,j))
                result (i,j) = result(i,k) + result(k,j);
            end
        end
    end
end

sum =0;
for i=1:length(x)
    for j=1:length(x)
        sum = sum+result2(i,j);
    end
end

avg_dist = sum/(n*n);

count = 0;
b=0;
connected=1;
for i=1:length(x)
    for j=1:length(x)
        if result (i,j) < n

```

```

        count = count +result (i,j) ;
        b=b+1;
    else
        connected=0;
        break;
    end
end
if connected==0 break; end;
end
if connected==1
    average_path = count/b;
else
    average_path = -1;
end

function [average_path avg_dist]= Short_Path3 (r,n)
x = rand(1,n);
y = rand(1,n);
z = rand(1, n);
result = zeros(length(x), length(x));
result2 = zeros(length(x), length(x));
for i=1:length(x)
    for j=1:length(x)
        if (i~=j)
            result2(i,j) = sqrt((x(i)-x(j))^2+(y(i)-y(j))^2 +(z(i)-z(j))^2);
            if (r > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2)+(z(i)-z(j))^2)
                result(i,j) = 1;
            else
                result(i,j) = 2000;
            end
        end
    end
end
end

for k=1:length(x)
    for i=1:length(x)

```

```

        for j=1:length(x)
            if result(i,j) > ( result(i,k) + result(k,j))
                result (i,j) = result(i,k) + result(k,j);
            end
        end
    end
end

sum =0;
for i=1:length(x)
    for j=1:length(x)
        sum = sum+result2(i,j);
    end
end
avg_dist = sum/(n*n);

count = 0;
b=0;
connected=1;
for i=1:length(x)
    for j=1:length(x)
        if result (i,j) < n
            count = count +result (i,j) ;
            b=b+1;
        else
            connected=0;
            break;
        end
    end
    if connected==0 break; end;
end
if connected==1
    average_path = count/b;
else
    average_path = -1;
end
end

```

```

function [average_path avg_dist]= Short_Path3 (r,n)
x = rand(1,n);
y = rand(1,n);
z = rand(1, n);
result = zeros(length(x), length(x));
result2 = zeros(length(x), length(x));
for i=1:length(x)
    for j=1:length(x)
        if (i~=j)
            result2(i,j) = sqrt((x(i)-x(j))^2+(y(i)-y(j))^2 +(z(i)-z(j))^2);
            if (r > sqrt((x(i)-x(j))^2+(y(i)-y(j))^2)+(z(i)-z(j))^2)
                result(i,j) = 1;
            else
                result(i,j) = 2000;
            end
        end
    end
end
end

for k=1:length(x)
    for i=1:length(x)
        for j=1:length(x)
            if result(i,j) > ( result(i,k) + result(k,j))
                result (i,j) = result(i,k) + result(k,j);
            end
        end
    end
end

sum =0;
for i=1:length(x)
    for j=1:length(x)
        sum = sum+result2(i,j);
    end
end
end

```

```

avg_dist = sum/(n*n);

count = 0;
b=0;
connected=1;
for i=1:length(x)
    for j=1:length(x)
        if result (i,j) < n
            count = count +result (i,j) ;
            b=b+1;
        else
            connected=0;
            break;
        end
    end
    if connected==0 break; end;
end
if connected==1
    average_path = count/b;
else
    average_path = -1;
end

```